

Final Exam (ECON300-1)

Fall 2002

Prof. Masami Imai

Name_____

Instructions: Show your work, reasoning, and calculations to receive full credit. I will not “guess” what your answers are, so please write legibly. The maximum score is 220 (224 if you get correct answer for the bonus question in the last page). Good luck!

1. True or False. Explain.

(a) (4 points) If event A and event B are mutually exclusive, then they are independent as well.

(b) (4 points) If $R = \log(X)$ where X is random variable, then $E(R) = \log(E(X))$.

(c) (4 points) If an estimator V_1 is a biased estimator of population parameter θ while V_2 is an unbiased estimator of θ , then it must be the case that V_2 is always more efficient than V_1 .

(d) (4 points) Multicollinearity problems occurs when two or more regressors are strongly correlated with error terms.

2. Chandler and his roommate Joey are thinking of getting flu shots. Suppose that (1) the chance of Chandler getting the flu is 0.3 while the probability of Joey getting the flu is 0.01 and (2) the chance of Chandler getting the flu if Joey gets the flu is 0.3.

(a) (4 points) Let C denote the event that Chandler gets the flu and let J denote the event that Joey gets the flu. Then, is C independent of J under the assumption (1) and (2)? Explain why or why not.

(b) (4 points) What is the probability that both Chandler and Joey get the flu?

(c) (4 points) What is the probability that neither get the flu?

(d) (4 points) What is the probability that at least one of them gets the flu?

3. Your friend Monica is known to be bad at bluffing when she plays poker. In particular, everybody knows that she plays with her hair 90 percent of the time when she has a good hand. Furthermore, she plays with her hair 2 percent of the time when she has a bad hand. Assuming that the (unconditional) probability of Monica having a good hand is 0.5, answer the following questions

(a) (4 points) If Monica starts playing with her hair after she was dealt her cards, what is the probability that she has a bad hand?

(b) (4 points) If she does not play with her hair, what is the probability that she has a good hand?

4. Professor Geller has 100 students in his class. His final exam consists of 10 true or false questions. Students get 1 point for each correct answer, and they must get at least 6 points to pass this class.
- (a) (4 points) Suppose that Phoebe, one of his students, goes into the exam without knowing a thing, and thus she must purely guess. What is the chance of her passing?
- (b) (4 points) Suppose that Chandler, another student of Dr. Geller, absolutely knows the correct answers for the first 5 questions while he has no clue on the remaining 5 questions. What is the chance of him passing?
- (c) (4 points) Suppose that all of Dr. Geller's students are exactly like Phoebe in that they must make guesses on every question. What is the probability that more than 50% of his students fail his class.

5. The return of Japanese stocks, X , is random and normally distributed with mean $\mu_j = 10\%$ and standard deviation $\sigma_j = 10\%$. The return of US stocks, Y , has the same expected return as Japanese stocks but a larger risk than Japanese stocks in that $\mu_{us} = 10\%$ and $\sigma_{us} = 11\%$. Answer the following questions assuming that the return of Japanese stocks is independent of the return of US stocks.

(a) (4 points) If you invest all of your money into Japanese stocks, then what is the expected return of your portfolio? What is the standard deviation of your portfolio's return?

(b) Suppose instead that you invest half of your money into Japanese stocks and the other half into US stocks.

1. (4 points) What is the expected return of your portfolio?

2. (4 points) What is the standard deviation of your portfolio's return?

(c) (4 points) Suppose that your investment advisor said, “since Japanese stocks give you the same expected return as US stocks while they are less risky, you should simply put all of your money into Japanese stocks.” What is your response to this advice in light of your answers in part (a) and (b).

(d) (8 points) In reality, the return of Japanese stocks and that of US stocks are not independent; they are somewhat positively correlated. Do (b)-1 and (b)-2 assuming that $\rho_{XY} = 0.70$.

6. Suppose that you are interested in estimating the population proportion of Republican voters, denoted by π . You will randomly choose 300 eligible voters and ask them to identify themselves as Republican voters or not. Using this data, you can calculate the sample proportion of Republican voters, denoted by P .

(a) (4 points) What is the expected value of P .

(b) (4 points) What is the standard deviation of P .

(c) (4 points) How is P distributed? Justify your answer by clearly stating the theorem that you used.

(d) (4 points) What is the probability that the interval $(P - 0.1, P + 0.1)$ will contain π .

(e) (4 points) After collecting data, it turns out $P = 0.60$. Construct a 95% confidence interval for π .

(f) (4 points) Interpret the 95% confidence interval calculated in (e) in words.

7. Suppose that you collect a random sample of size n from a population that has a mean μ and a standard deviation σ . Consider the following estimator of population mean μ :

(a) $\hat{X} = \frac{1}{n+1} \sum X_i$

1. (4 points) Calculate $E[\hat{X}]$. Is this estimator unbiased?

2. (4 points) Calculate $Var[\hat{X}]$.

3. (4 points) Is this estimator consistent? Show your calculation.

8. Suppose a random sample of 3 pharmaceutical firms had the following profits, Y , and patents, X .

Observation	Y (Profit in million dollar)	X (number of patents)
1	2	1
2	3.4	2
3	6	3

- (a) (4 points) Calculate the sample mean of Y and the sample mean of X (\bar{Y} and \bar{X}).

- (b) (4 points) Calculate the sample variances of Y and X (S_Y^2 and S_X^2).

- (c) (4 points) Calculate the sample correlation coefficient, r , between X and Y . What does it tell us about the relationship between profits and patents?

- (d) (4 points) Calculate the estimates of α and β in the regression model below:

$$Y_i = \alpha + \beta X_i + e_i.$$

Even if you are not able to answer this question, then you should still do the rest of questions, assuming that $a = 0$. and $b = 1.9$.

- (e) (4 points) According to the estimated regression equation in (d), how much profit would three additional patents generate for a firm.

- (f) (4 points) Calculate the unexplained sum of squares and the total sum of squares.

- (g) (4 points) Calculate the coefficient of determination, r^2 , by taking the ratio of unexplained SS to total SS. Interpret it in words.

- (h) (4 points) Predict the profit of a randomly drawn firm that has 2 patents by constructing a 95% confidence interval around the predicted value (assume that the standard error of regression, s , is 0.24).
- (i) (4 points) You wish to test the claim that profits are positively associated with the number of patents. State the null and alternative hypothesis in symbols.
- (j) (4 points) Compute p-value associated with the null hypothesis (assume again that the standard error of regression, s , is 0.24). Is the null hypothesis credible?

9. Suppose that Dr. Geller took a survey on weekly wage income, sex, and years of schooling from randomly chosen 325 individuals, all of whom are 40 years old and full-time workers.

(a) First of all, he used Excel and obtained the following descriptive statistics of wages for men and women.

<u>Men's Weekly Wages</u>		<u>Women's Weekly Wages</u>	
Mean	584	Mean	393
Standard Error	33	Standard Error	20
Median	516	Median	317
Mode	577	Mode	231
Standard Deviation	429	Standard Deviation	254
Sample Variance	183868	Sample Variance	64569
Kurtosis	21	Kurtosis	3
Skewness	4	Skewness	1
Range	3644	Range	1601
Minimum	42	Minimum	20
Maximum	3686	Maximum	1621
Sum	99925	Sum	60462
Count	171	Count	154

It is clear that the average wage of men (\$584) is larger than the average wage of women (\$393) by \$191 in this particular group of individuals. However, Dr. Geller is not sure if that is the case for the entire population.

1. (4 points) Construct a 95% confidence interval for the difference in the average wage between men and women, assuming that the population variance of men's wages is the same as the population variance of women's wages. Is the difference in average wages between men and women statistically discernible at a 95% confidence level.

2. (4 points) Construct a 99% confidence interval for the difference in the average wage between men and women, assuming that the population variance of men's wages is the same as the population variance of women's wages. Is the difference in average wages between men and women statistically discernible at a 99% confidence level.

3. (4 points) Based on the previous answers, how sure can Dr. Geller be in claiming that the average wage of men is greater than the average wage of women in the population.

4. (4 points) Based on the previous answers, how sure can Dr. Geller be in claiming that the average wage of men is greater than the average wage of women in the population because of gender discrimination. Explain your reasoning.

(b) Dr. Bing claimed that the observed wage difference of \$191 between men and women simply reflects the difference in the education level of men and women. Dr. Geller wanted to explore this possibility by estimating the following equation:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 SEX_i + e_i$$

where

$$\begin{aligned} Y_i & : \text{ weekly wage income of individual } i \\ X_i & : \text{ years of schooling of individual } i \\ SEX_i & = \begin{cases} 1 & \text{if individual } i \text{ is female} \\ 0 & \text{if individual } i \text{ is male} \end{cases} \end{aligned}$$

Eviews gave him the following result:

Dependent Variable: Y
 Method: Least Squares
 Date: 12/08/02 Time: 11:02
 Sample: 1 325
 Included observations: 325

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	27.66981	83.58151	0.331052	0.7408
X	42.70673	6.107131	6.992929	0.0000
SEX	-206.0521	37.05371	-5.560903	0.0000
R-squared	0.190469	Mean dependent var	493.4988	
Adjusted R-squared	0.185440	S.D. dependent var	368.9978	
S.E. of regression	333.0314	Akaike info criterion	14.46354	
Sum squared resid	35712997	Schwarz criterion	14.49847	
Log likelihood	-2347.325	F-statistic	37.88048	
Durbin-Watson stat	1.261269	Prob(F-statistic)	0.000000	

1. (4 points) Given this estimation result, the predicted weekly wages are:

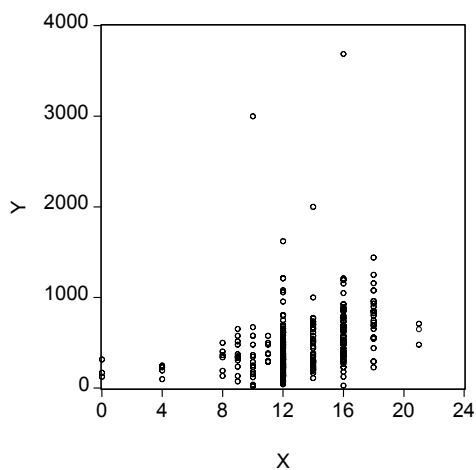
$$\hat{Y} = 27.7 + 42.7X - 206.1SEX.$$

Draw a graph that exhibits how wages are related to the years of schooling for men and women (hint: you need to draw two lines in your graph, one line for men and the other for women). Make sure that both horizontal and vertical axes are labeled.

2. (4 points) Interpret the coefficient on X and SEX in words.

3. (4 points) Does the data support Dr. Bing's initial claim that the wage difference between men and women simply reflects the difference in education level? Explain your answer in words.

- (c) After having estimated a linear equation, Dr. Geller is wondering whether he needs to use a nonlinear model. He produced a scatter plot of weekly wage (Y) against the years of schooling (X) as follows.



1. (4 points) Does the graph exhibit that wage is positively related to education as suggested by the estimation results in (b)?

2. (4 points) Write down the appropriate nonlinear function that relates Y to X ? Is it linear in parameter? If not, how do you transform it so that you can use an ordinary least square method to estimate this nonlinear relationship between X and Y .

10. Are multinational corporations good for indigenous workers? Lipsey and Sjöholm (2001) investigated whether foreign firms pay higher wages than domestic firms while keeping constant the characteristics of workers in Indonesia. In particular, they used survey data from 19,579 firms in Indonesia and estimated the following regression equation:

$$\log(WAGE_i) = \beta_0 + \beta_1 PRIMARY_i + \beta_2 JUNIOR_i + \beta_3 SENIOR_i + \beta_4 UNIV_i + \beta_5 GOV_i + \beta_6 FOREIGN_i + e_i$$

$\log(WAGE_i)$	log of average wage in firm i
$NOPRIM_i$	share of employees with <i>below</i> primary education in firm i
$JUNIOR_i$	share of employees with junior education in firm i
$SENIOR_i$	share of employees with senior education in firm i
$UNIV_i$	share of employees with university education in firm i
GOV_i	1 if firm i is owned by government and 0 otherwise
$FOREIGN_i$	1 if firm i is owned by foreigners and 0 otherwise

- (a) (4 points) How do you interpret β_6 in words? Hint 1: if firm i is privately owned by Indonesian individuals, then $GOV_i = FOREIGN_i = 0$. Hint 2: the dependent variable is a log of average wages, not average wages.

- (b) (4 points) How do you interpret β_1 in words? Hint 1: $NOPRIM$, $JUNIOR$, $SENIOR$, and $UNIV$ are included to control for the quality and skill-level of employees in regression analysis. Hint 2: if all of employees in firm i have only primary education, then $NOPRIM_i = JUNIOR_i = SENIOR_i = SENIOR_i = 0$. Hint 3: the dependent variable is a log of average wages, not average wages.

(c) (4 points) What do you expect the signs of β_1 and β_2 to be. Explain.

(d) (4 points) The researchers obtained the following estimation results:

The dependent variable is $\log(\text{WAGE})$

<i>Regressor</i>	<i>Coefficient (standard error in parenthesis)</i>
NOPRIME	-0.48 (0.033)
JUNIOR	0.41 (0.022)
SENIOR	0.68 (0.019)
UNIV	1.85 (0.15)
GOV	0.18 (0.036)
FOREIGN	0.36 (0.023)
Number of observations	19579
Adjusted R-squared	0.18

Note that the coefficient on FOREIGN is positive, indicating that foreign firms on average pay higher wages than domestic firms, given an education level of employees. However, can we reject, at a 95% confidence level, the null hypothesis that foreign and domestic firms pay the same wages to workers given the same education level.

- (e) Suppose hypothetically that the researchers had not included *NOPRIM*, *JUNIOR*, *SENIOR*, and *UNIV* in the right hand side of the regression equation.
1. (4 points) Would adjusted R-squared have been larger without these explanatory variables? Explain.
 2. (4 points) Do you think the coefficient on *FOREIGN* would have been larger or smaller without these explanatory variables? Explain.
- (f) (8 points) Why do you think foreign firms end up paying higher wages than domestic firms? How would you expand Lipsey and Sjöholm's regression equation to test your hypothesis? In other words, what additional quantifiable variable would you include in the right-hand side of the equation.

11. (4 bonus points) *This is a bonus question and no partial credit is awarded for this question. Be also aware that it is extremely difficult!! You should do this problem only if you are absolutely sure that there is nothing you can do to get more points from the other questions!! Note that you only get 4 points for this question despite its difficulty!!*

Suppose that the price of oil (denoted by X) is a normally distributed random variable and that its mean and standard deviation are \$3 per gallon and \$1 per gallon, respectively. Suppose also that the demand for oil, Y , is a linear function of X as follows: $Y = 100 - 5X$. Note that $I = XY$ (price times quantity) is the total expenditure on oil. Compute $E(I)$.